Optimal control of long-term sentimental dynamics

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November 30, 2011

1 Introduction

This contribution is concerned with love dynamics. In particular, we consider committed relationships intended to endure. Given the current grim statistics about divorce, a fundamental problem consist of understanding what makes some couples successful while others are miserable. This is an old question already posed by psychologist Louis Terman in the 1930s [1].

A further contemporary problem arises from sociological evidence. On one hand, there is an epidemic of failure among couple relationships in the western world. This is evidenced in many surveys and studies, e.g. [**?**]. Also, there is evidence that most people consider a long-term relationship as a main ingredient of happiness and, furthermore, most of them would consider their own relationship stable [4, 5]. These two facts together pose what can be called the *failure paradox:* how is it that, while most couples are ready to commit to a lasting happy relationship, a massive proportion of them will fail?

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This note seeks to contribute to understand the problem of long-term sentimental relationships via a mathematical model based in optimal control theory.

2 The (Adam and Eve) model

The model makes three simple assumptions, that may be considered plausible in the case of Adam and Eve's love story.

The first assumption is *homogamy*: the couple itself is the planning unit. In particular, no interaction dynamics within the couple is considered. This strong assumption amounts to considering the partners as similar. This seems valid in the West, where people choose their long-term partners among those who are similar to them (see e.g. [6]). A weaker form is similarity is needed here: partners share the parameters and functional structure defined below.

The second assumption is the *second law of thermodynamics of marital relationship*: there is a natural decay of the sentimental dynamics, that can be counteracted with effort. This fact is widely recognized by scholars and therapists and it was formulated as a law by Gottman et al [1]. The second law can be written as a linear differential equation that describes the decay of the variable $x(t) \geq 0$ describing the state of the relationship (the *feeling*) plus a recovery term that comes from the effort making, whose level is given by the variable $c(t) \geq 0$ (the *effort*). The equation reads

$$\frac{dx}{dt} = -rx + ac, \tag{1}$$

where $r > 0$ measures the strength of the feeling fading inertia and $a > 0$ represents effort efficiency.

The third assumption concerns *well-being valuation*: couples evaluate their relationship through a cost-benefit approach. Feeling is something good that produces satisfaction but in a decreasing fashion. So feeling satisfaction is described by some increasing and concave function $U(x)$ that goes flat when feeling becomes large, i.e. $U'(x) > 0$, $U''(x) < 0$, and $U'(x) \to 0$ as $x \to +\infty$. Bad things, however, escalate. Since high levels of effort are considered a bad thing, effort discomfort $D(c)$ is assumed to be increasing and convex beyond certain level $c^* \geq 0$. However, effort levels below c^* may have a positive effect on well-being, so that partners enjoy making the effort provided it is not excessive. It is thus assumed that $D''(c) > 0$, $D'(c^*) = 0$, and $D'(c) \to +\infty$ as $c \to +\infty$.

The assumptions above allows us to formulate the couple's plan for their relationship as an optimal control problem: the couple determines the effort path that makes them as happy as possible and such that the entailed feeling-effort dynamics is sustainable in the long-term. Total happiness is expressed as the aggregate net well-being during the non-terminating life of the relationship, i.e.

$$\int_0^\infty \exp(-\rho t)\,(U(x) - D(c))\,dt.$$

The instantaneous satisfaction is balanced with an exponential term that represents temporal preference in a standard way.

3 Results

Optimal control theory [7] produces the fundamental equation for the optimal effort dynamics:

$$\frac{dc}{dt} = \frac{1}{D''(c)}\left((\rho + r)D'(c) - aU'(x)\right). \tag{2}$$

It is apparent from above that the favorite effort level c^* is never a solution of the couple's problem.

- *The back room analysis.* The analysis of the coupled feeling-effort dynamics (1)-(2) produces significant features. First, there is a unique sentimental equilibrium $E = (x^\heartsuit, c^\heartsuit)$. E is admissible if $x^\heartsuit > x_{\min}$, where $x_{\min}$ is a certain threshold value below which the relationship is not considered satisfactory.

  A key finding is that the required effort at equilibrium $c^\heartsuit$ is always bigger than the effort level preferred a priori c^*. The existence of this effort gap represents an unexpected difficulty for the viability of the relationship: even if the feeling is rewarding, E may not be viable if the size of the gap is important.

  The global dynamics determines the evolution of the sentimental dynamics, which is typically expected to start at a high feeling level $x_0 >> x_{\min}$. The phase space turns out to be that of a nonlinear saddle. In particular, the feeling-effort dynamics is unstable. Thus, a

long-term sustainable dynamics is only possible if the initial effort $c(0)$ is fine-tuned to get onto the stable manifold W^- –say at a point A– and then the trajectory of A, $\gamma_A \subset W^-$, is followed until E is reached. Other trajectories are either not admissible or lead to feeling extinction. Again there is an effort gap along γ_A: the required effort level to make the transition from A to E is always above c^*. Thus, lasting relationships are possible: if E is rewarding enough, if the effort gap is tolerable, if instability is resisted along the way to E, if the couple is alert to watch the correct effort permanently.

- *The front room analysis.* Things look different from the front room, where real relationships take place. While lasting relationships are possible they are somehow exceptional. A successful long-term story seems very demanding, namely: equilibrium must be found and the problem solved to find $\gamma_A \subset W^-$ –and there is only one such successful trajectory–; the equilibrium E must be viable, i.e. $x^\heartsuit > x_{\min}$; effort stress must be tolerable along the path γ_A; effort instability must be resisted along γ_A; and the effort gap that still persists if E is reached must remain monitored.

- As a consequence, there are apparent reasons not to be optimistic about the future of a relationship in the front room.

4 Conclusions

Some key sources of instability revealed by optimal control theory explain the endangered future of a committed relationship. The analysis suggests a general robust mechanism that may be acting behind the breakdown of many couple relationships in realistic scenarios. It is the combination of two effects –the effort gap plus the instability of the dynamics–, provoking that when effort is relaxed, the feeling is driven far from the successful path towards non-admissible levels.

The theory also offers a possible explanation for one mystery in the field of marital therapy [1]: the fact that couples that went through therapy relapse after some time. The model suggests that, if therapy restores the state of things back to the stable path, instability and the effort stress may operate together to take the relationship out of the successful track again after some time.

The model study also produces a mechanistic explanation of the failure paradox, as a consequence of the couple's planning behavior and the second law. The theory so gives meaning to a paradoxical statement: Our love was to last forever. Hence we are now breaking up.

References

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