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Ecological Modelling 182 (2005) 221–228
ECOLOGICAL MODELLING
www.elsevier.com/locate/ecolmodel

An entropy-based heterogeneity index for mass–size distributions in Earth science

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Abstract

A quantitative classification of soil texture is proposed based on an entropic index that can be easily computed from knowledge of the fractional contents of soil textural classes. It is first shown that the index formula supplies a number that agrees with the entropy dimension when the corresponding soil particle-size distribution (PSD) displays self-similar fractal features. In the absence of self-similarity, the index is further shown to retain information-theoretic content so that it becomes a meaningful diversity index in the general case. The index is defined by balancing Shannon's entropy in an appropriate way to deal with the high variability of the interval lengths used to report soil particle size classes. The performance of the proposed formula is illustrated for standard textural data reported as clay–silt–sand soil mass fractions. The index induces a classification of a continuum of textural classes that may distinguish soils within the same standard textural class, thus establishing a continuous characterisation of textures that is complementary to the usual classification, but requires no additional information. Finally, it is shown how the balanced-entropy index might also be used as a measure of body size diversity for living organisms.

Keywords: Soil texture; Biodiversity; Shannon's entropy; Information theory; Fractal geometry

1. Introduction

The classification of soil and sediment textures plays an important role in the Earth sciences. In particular, the statistical description of soil particle-size distributions (PSD) is of great importance in the study of soil physical properties.

The usual classification of textures defines textural classes grouping together soils with mass percentages of clay, silt and sand between certain prescribed limits. Different classifications of soil textures have been proposed (Folk, 1954; Shepard, 1954; Baver et al., 1972; Soil Conservation Service (SCS), 1975; Vanoni, 1984). These systems differ both in the particle-size limits chosen to separate the size groups and in the percentage limits established to define each textural class.

Since many different combinations of clay, silt and sand may correspond to the same textural class, soil samples of rather diverse composition appear indistinguishable under the grouping that these classes establish. Shirazi and Boersma (1984) proposed a classification based on the addition of new information to the conventional texture triangle used by the United States Department of Agriculture (USDA). By integrating on the textural triangle geometric means and standard deviations obtained from mechanical analysis of soil samples, they derive a new diagram which provides greater resolution in detecting

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doi:10.1016/j.ecolmodel.2004.04.002

classified soil samples within a textural region. However, because of its character of first approximation to PSD, a classification system of textures should keep a trade-off between simplicity and taxonomic power. A more efficient statistical description may thus be inadequate for classification purposes if it is achieved at the cost of obtaining extra non-trivial information.

The issue is whether the characterisation of soil textures can be refined and unified without requiring any further information than that employed by any of the standard classification systems, e.g. by the USDA or the International Society of Soil Sciences (ISSS), that is, data of soil mass percentages of primary particles only.

The goal of this note is to propose a uniparametric continuous characterisation of textures by means of an index built from Shannon's entropy (Shannon, 1948a,b) that can be computed from soil mass fractions of primary particles. The use of the index as a textural parameter arose from a fractal modelling for PSD (Martín and Taguas, 1998). Under the fractal model, the index is the so-called *entropy dimension* of the underlying fractal distribution which in turn yields rich information on the scaling behaviour of mass distribution with respect to particle sizes. However, if the fractal model is not assumed to describe the distribution of soil particle masses, the index can still be understood from information theory and can be shown to carry information about the heterogeneity of a PSD.

The ideas above also appear to be useful in ecology, namely for evaluating diversity of body size distribution in living organisms, which is a problem remarkably having common features with that of evaluating PSD textural heterogeneity. However, further biodata work is needed to illustrate the use of the index in this context.

The use of the index for practical classification of textures stems from (Martín and Taguas, 1998; Martin et al., 2001) and its role for measuring body size diversity was addressed in (Martín and Rey, 2002). The general ideas behind the theoretical framework are discussed in Sections 2 and 3. In Section 4, a practical study using clay, silt and sand percentages corresponding to 171 real soil data from Soil Conservation Service (SCS, 1975) is performed to show the ability of the proposed parameter to characterise soil textures. Section 5 comments on the possible use of the balanced-entropy index to evaluate body size diversity.

2. Models and parameters for texture that require no extra information

The challenge is to find relevant parameters, maybe through suitable models, to characterise PSD without requiring any more information than that supplied by usual textural data. Assume for the sequel that textural data are supplied by the fractions (P_1, P_2, P_3) of the mass of soil particles with characteristic sizes respectively within the intervals I_1, I_2, I_3, which are prescribed to report textures. The basic choice $I_1 = [0, 0.002]\,(\text{mm})$, $I_2 = [0.002, 0.05]\,(\text{mm})$ and $I_3 = [0.05, 2]\,(\text{mm})$ used by the USDA classification will be considered in this paper. It may be noted that any other choice, varying either the number or the size of the intervals, may be considered within the scheme of the model and the accompanying parameter described below.

Under the point of view of the statistical description of PSD, infinitely many different distribution models may be conceived to fit given textural data (P_1, P_2, P_3), even under strong assumptions like, for instance, log-normality. Of course, each one of them would predict differently the distribution of mass inside the intervals I_1, I_2, I_3, when nothing is known from the given data. The selection of a best model to describe the real distribution would require extra data on particle sizes at a finer resolution than those reported by I_1, I_2, I_3.

However, for classification purposes, the problem has further subtle shades, since the little amount of textural information, as reported above, is all that one has to design a parameter that differentiates textures. Such a parameter should ideally capture some meaningful feature of the PSD, rather than describing the entire distribution.

The approach of the fractal model below links in fact both aspects. First, a distribution model, unequivocally determined from usual textural data without extra information, is constructed to fit the data and to replicate unknown data at smaller size scales. Second, the model provides a easily computable parameter which proves relevant for the characterisation of soil PSD's.

2.1. A fractal model for PSD

PSD may be thought of as a mass distribution in the interval $I = [0, 2]\,(\text{mm})$ of particle sizes assigning to each interval of sizes $[a, b]$ the mass of soil particles whose sizes are between *a* and *b*. A key feature of PSD heterogeneity is the wide disagreement or absence of any proportionality between the length of any of the three basic size intervals ($I_1 = [0, 0.002]$, $I_2 = [0.002, 0.05]$, $I_3 = [0.05, 2]$) and the masses of soil particles with characteristic sizes within those intervals. As a matter of fact, scale invariance in the PSD distribution, which is strongly suggested from real data analysis, indicates that this disagreement may hold within a range of scales. In Martín and Taguas (1998), it is assumed that it occurs at every scale leading to a fractal model for PSD based on self-similar distributions. These sorts of distributions, intensively studied in fractal geometry, satisfy that the mass distribution on the basic intervals I_1, I_2, I_3 is reproduced within each one of them (suitably rescaled) and it is again reproduced within each one of the rescaled basic intervals within them, and so on. In this way, a fractal self-similar distribution *M* is obtained within the interval [0, 2] which matches the textural data, assigning the right mass to each one of the basic intervals, and it also replicates this mass distribution structure within smaller size intervals. Formally, *M* is constructed from a set of linear mappings and accompanying probabilities, i.e. an iterated function system. The model building is very flexible in the sense that a distribution *M* can be constructed from any number of size classes (three in the USDA system), each of any length. The lengths of the classes may and do vary depending on national classifications, available data, etc. For instance, in the ISSS classification system, the three basic size classes defined by $I_1 = [0, 0.002]$ (clay), $I_2 = [0.002, 0.02]$ (silt), $I_3 = [0.02, 2]$ (sand) are employed. The details on the PSD model and the supporting theory can be seen in Martín and Taguas (1998). The good performance of the fractal model when compared to real data is shown in Fig. 1.

The idea behind this model is that the heterogeneity displayed by the mass percentages corresponding to the different size fractions is not a feature observed at a privileged scale only, but it also occurs within a range of (smaller) scales in a similar manner. This is a simple hypothesis among all plausible ones in order to formulate a conjecture for the complex behaviour of PSD at unobservable scales. Self-similarity is a natural assumption that squeezes the a priori poor information carried by the three single percentages of textural data to determine the entire PSD distribution.

The capability of the model to describe the PSD of a given real soil was considered and discussed

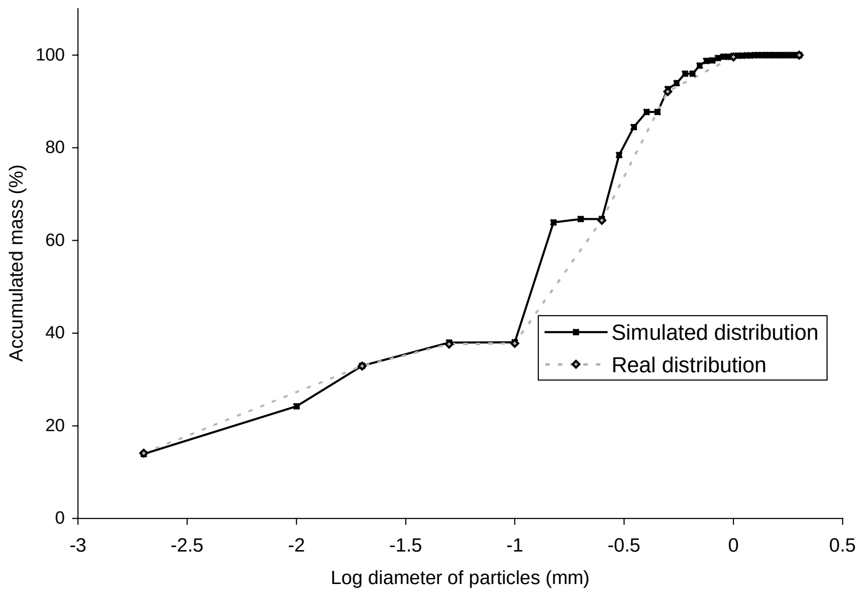

Fig. 1. From the knowledge of the mass fractions supported by the particle-size intervals [0, 0.1], [0.1, 0.25], [0.25, 2], 42 intermediate values are simulated using the fractal model (Soil Horizon A12, p. 498, SCS, 1975).

in detail in Taguas et al. (1999). In that work, the descriptive power of the model was strongly evidenced from the analysis of a large class of soils, thus conferring validity on the self-similarity approach to be further exploited while keeping, at the same time, the need for additional information to a minimum.

2.2. A new parameter for texture: entropy dimension

Soil PSDs are heterogeneous intricate distributions which reveal new information when scrutinised at smaller and smaller scales and thus theoretical parameters coming from measure theory and information theory may conceivably be adapted to describe the complexity of these sorts of particular distributions. Shannon's entropy (1948a,b) is an information-theoretical parameter that may be suitably interpreted as a measure of the complexity of a distribution. In fact, entropy has already been proposed in the life sciences as a plausible measure of biodiversity (Margalef, 1958) in the sense of evenness or heterogeneity of the diversity of species in an ecosystem. The same use of entropy to measure pedodiversity has been recently discussed (Ibañez et al., 1998; Martín and Rey, 2000; Ibañez and De-Alba, 2000). Apparently, entropy has not been proposed so far in the Earth sciences as a measure of soil texture or sediment particle size heterogeneity.

In the context of this note, if the size interval is partitioned into many size intervals $J_1, J_2, \ldots, J_k$ of characteristic length r and with soil particle mass fractions $p_1, p_2, \ldots, p_k$, respectively, the Shannon's entropy of the partition, which is defined by

$$H(r) = -\sum_{j=1}^{k} p_j \log p_j,$$

is expected to increase without bound as the length scale r decreases ($r \to 0$ or $k \to \infty$). This is because the complexity of the PSD increases when the grain size resolution magnifies. This leads to the computation of the growth rate of $H(r)$ with respect to the scale: the number D such that

$$H(r) \propto -D \log r,$$

is called the *entropy dimension* of the distribution (Renyi, 1957). Entropy dimension is a natural and theoretically founded parameter linked to the degree of heterogeneity of complex distributions and it becomes in turn a candidate for a fine quantitative characterisation of individual soil textures.

In soil sciences, it has been recently shown that the entropy dimension can be properly defined from real data for grain size soil distributions (Martín and Montero, 2001) and pore size soil distributions (Caniego et al., 2001). However, fair direct computations of entropy dimensions, such as those mentioned above, require textural data for a wide range of scales, which are not usually available from standard data. In principle, this implies that entropy dimension, being theoretically well adapted to measure texture, would be useless for practical purposes.

However, the fractal model for PSD described in Section 2.1 can be used to overcome the difficulty in computing entropy dimensions discussed in the preceding paragraph. Indeed, the assumption of the model plus theoretical results from fractal geometry show that, for a self-similar PSD, the entropy dimension is given by the simple formula

$$D = \frac{\sum_{i=1}^{3} P_i \log P_i}{\sum_{i=1}^{3} P_i \log r_i},$$

where the P_i's are the soil's clay–silt–sand fractions and the numbers r_i are the ratios between the lengths of the three basic size intervals I_i and the length of the size interval I (that is, 2). In the USDA system, $r_1 = 0.001$, $r_2 = 0.024$ and $r_3 = 0.975$. The formula above can be easily computed from conventional textural data and it thus provides an efficient and straightforward way of evaluating the entropy dimension of the fractal model replicating PSD from textural data. The entropy dimension D takes values between 0 and 1, and it may be interpreted as follows: the higher the value of D the more heterogeneous the soil's PSD and in turn the richer the soil's textural structure. Moreover, since D may take any value from 0 to 1, entropy dimension—supplying a continuum of textural classes—adds a further criterion of discrimination of soil textures in terms of heterogeneity, when compared with standard classifications. A thorough interpretation of the above formula and its theoretical properties in terms of texture analysis is given in Martín et al. (2001).

2.3. Balanced entropy as a diversity index without any underlying model

A main objection for the use of the entropy dimension D as a soil textural heterogeneity index might be the assumption of the self-similar PSD model to which the interpretation of D is tied. Strictly speaking, the formula for D has the meaning explained in Section 2.2 as far as the self-similarity of PSD (respect to the similarities defined through the basic partition) is guaranteed. The closer a PSD is to the self-similar model, the closer its entropy dimension is to the index D. Although in Taguas et al. (1999) the fractal model was consistent with data for a large number of soils, the self-similar hypothesis may not be assumed to hold in general.

Notice, however, that the entropy dimension formula can be naively computed from textural data for any soil independently of the degree of self-similarity which is assumed. Thus, the question arises whether the numbers D so obtained still carry any information that may be linked to textural heterogeneity in some sense. This release from the underlying fractal model would permit the use of D as a heterogeneity indicator for arbitrary soil textures. It is actually the case that D can be interpreted in this way within the framework of information theory.

Remember that the parameters r_i were defined as the ratios between the length of the basic size interval I_i and that of the size interval I, which is 2. Notice that $r_1+r_2+r_3=1$ so that the size parameters r_i's define, as the masses distribution P_i, a probability distribution in the interval of textures I. It turns out that the number D satisfies (Martín and Rey, 2002)

$$D=\frac{H}{H+d(P_i||r_i)}$$

where H is the Shannon's entropy of the mass distribution and $d(P_i\|r_i)$ is the *Kullback–Leibler* (KL) *distance* (Kullback and Leibler, 1951) between the distributions of masses P_i and the distribution of sizes r_i (which is given by $d(P_i||r_i)=\sum_i P_i\log(P_i/r_i)$; in this case, see Cover and Thomas, 1991). This index is called *balanced entropy* by Martín and Rey (2002), where it is proposed for evaluating diversity, in the sense of evenness or heterogeneity, of a general mass–size distribution defined using any number of size intervals with arbitrary lengths. In that paper, it is justified that balanced entropy generalises Shannon's entropy H as an heterogeneity index and that it must be used in place of Shannon's entropy to consistently evaluate a distribution's heterogeneity when reporting mass proportions on size intervals of quite unequal length.

Properties of balanced entropy that are of significance for the problem of measuring heterogeneity are mentioned next. As a consequence of the information inequality (Cover and Thomas, 1991), the index D takes values in [0, 1]. The case $D=0$ just occurs for the most uneven or homogeneous distribution, since all the mass is carried by one size class only. On the other hand, $D=1$ if and only if $P_i=r_i$, which corresponds with the most even or heterogeneous distribution, because the mass is uniformly scattered within each size class. The size distribution r_i plays the role of a reference distribution of maximal heterogeneity with respect to the basic size partition. In the USDA system, the maximal heterogeneity texture corresponds to the fractions 0.1% clay, 2.4% silt and 97.5% sand, located very close to the rightmost corner of the textural triangle. Of course, the uniform sand class at the right vertex of the textural triangle has null balanced entropy, what gives account of the extremal homogeneity of this uniform texture—the whole mass being supported by only one class. For practical purposes of comparison of textures, the fact that totally homogeneous textures ($P_i=1$ for some i) are characterised by $D=0$ does not seem relevant: while there is no need for a parameter to discern among those textures, D does play a role in parametrising (comparing) textures with positive fractions of each class of primary particles, that is, when dealing with textures properly inside the textural triangle.

Notice that balanced entropy is simply a multiple of the entropy of Shannon in the case that the r_i are all equal. In general, the index D distorts Shannon's entropy to take into account the disparity of the lengths of the size intervals. In contrast with Shannon's entropy, D takes values in [0, 1] for any number N and any sizes of the intervals I_i. This makes it possible to use D to compare data with very different formats (i.e. N and r_i varying) in the sense that D always supplies a sort of distance measure to the reference distribution that satisfies $P_i/r_i=1$. In the context of soil, this permits comparing PSD data from different classifications that define a priori incomparable textural classes.

3. Using the entropy index

In this section, the use of the entropy formula is discussed in characterising soil texture from standard data reporting only the basic (clay, silt and sand) mass soil fraction content. Textural data of 171 soils were analysed that correspond with the two first horizons of soils described by Soil Conservation Service (SCS, 1975, pp. 486–742). Data from the USDA system are employed in the case study below, although the same conclusions are derived using the ISSS classification system.

As explained above, the size parameters r_i enter the entropy formula as $r_1 = 0.001$, $r_2 = 0.024$ and $r_3 = 0.975$ for the USDA system. The entropy dimension of any soil data is then computed by plugging the soil mass fractions P_1, P_2, P_3 that correspond with each size interval I_1, I_2, I_3 into the formula for D proposed by Martín and Rey (2002).

The entropy dimensions obtained for the considered soils are represented in the conventional textural triangle (Fig. 1). Regions defined by the analysed soils with entropy dimensions D below 0.2, between entropy dimensions 0.2 and 0.4, between 0.4 and 0.6 and between 0.6 and 1 are displayed in Fig. 2.

It may be seen that the regions are well-defined within the textural triangle, do not overlap and, when combined, tile the whole region covered by the soils considered. As expected from the fact that D depends continuously on the fractions P_i, soils with similar mass fraction distributions have close entropy dimensions, as shown within each dimensional region represented in Fig. 2. Also, it may be noticed that D increases with increasing percentages of sand particles. This is because the sand interval (I_3) is the biggest and it so contains the widest variety of size particles. As a result, soils with a heavier mass content in the sand fraction have richer textural compositions, which amounts to larger entropy dimensions.

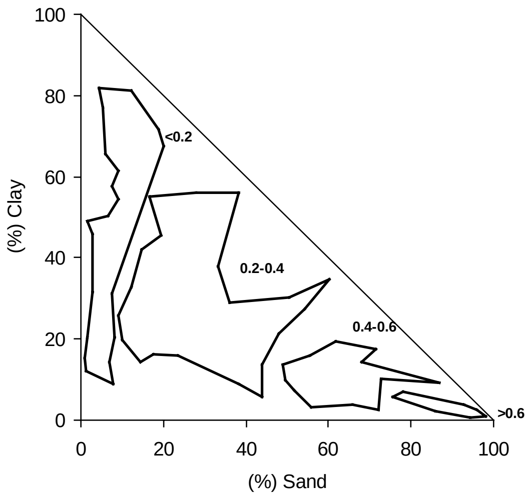

Fig. 2. Regions defined in the textural triangle considering several entropy dimension bounds using data of 171 soils from the USDA system.

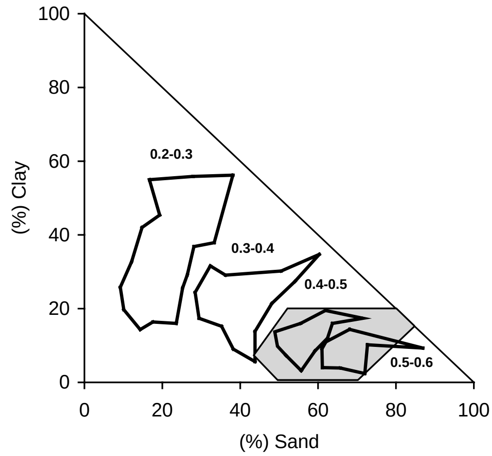

Fig. 3. Subregions defined in the textural triangle considering a finer partition of the value range [0, 1] for entropy dimensions using data of 171 soils from the USDA system. The boundary of the conventional sandy loam category is displayed to show that D discriminates soils within such a class.

It is also noticed that the entropy dimension, yielding a continuum of textural classes, is capable of discriminating soils inside the same textural class according to the usual classification. This can be appreciated in Fig. 3, where connected non-overlapping subregions containing soils with D between bounds 0, 0.2, 0.3, 0.4 and 0.5 are displayed inside the primary regions of Fig. 2.

4. A claim for the use of balanced entropy to evaluate body size diversity

The ideas developed above for the problem of heterogeneity of textures may notably apply in the field of ecological diversity. The reason is that the key features of PSD are matched *mutatis mutandis* by significant mass–size distributions in biodiversity, specifically by the distribution of biomass with respect to body sizes of living organisms within an ecosystem. Biomass diversity plays a significant role among the multiple aspects of ecological diversity.

Shannon's entropy has been used to measure biomass diversity (Lurié et al., 1983), using body size classes of uniform length regardless of their species composition. In this context I denotes the interval of body sizes, and the target distribution P_i describes the fraction of biomass carried by individuals whose body size is within the size interval I_i.

The crucial remark is that, as in the case of reporting soil texture, the use of uniform size partitions to processing real biomasss data seems to be highly non-efficient. This may be seen as a consequence of allometric laws (e.g. Damuth, 1981, 1987; Tokeshi, 1993), i.e. $N(W) \propto W^{-x}$, where x is a constant and $N(W)$ denotes the number of individuals with body size greater than W.

Remarkably, a similar scaling behaviour holds for PSD (Turcotte, 1986), i.e. $N(R) \propto R^{-d}$, where $N(R)$ is the number of particles of sizes greater than R and d is a constant. Since the (bio)mass range has such an enormous variation, a huge disproportionality occurs between the total biomass carried by a size interval and the body size range that the interval covers. This feature can be also grasped in models of biomass distribution (Lurié et al., 1983). As a consequence, biomass field data are expected to be reported using body size partitions of widely varying interval lengths r_i—probably differing in several orders of magnitude, whereas the corresponding biomass fractions P_i are comparable. In these cases, plain Shannon's entropy is not a sensible candidate to evaluate heterogeneity. We claim that the balanced-entropy index introduced in Section 4 may deal with such non-uniform size partitions while retaining the theoretical and practical properties that make Shannon's entropy suitable for evaluating biodiversity. Further work with real biomass data is in progress to investigate the role that may be played by balanced entropy in conservation ecology.

5. Conclusions

A simple index, obtained by balancing Shannon's entropy, is proposed to give a quantitative characterisation of textural heterogeneity of soil particle size distribution (PSD). The index is easily computable by a simple formula defined in terms of the standard clay–silt–sand ratios, no matter which system is employed to report soil texture.

Assuming a fractal model for PSD, theory from fractal geometry implies that the proposed parameter coincides with the entropy dimension of the model distribution.

In the absence of the fractal model, the number rendered by the formula can be understood from the point of view of information theory. In this general case, balanced entropy is a well-founded heterogeneity index that gives a sort of distance of PSD's to a fixed referential distribution.

The index induces a complementary continuous textural parametrisation that permits comparing soil textures reported using different standard systems.

The formula is computed for a large number of soils and the induced textural classification is represented inside the conventional textural triangle, showing how two different PSD heterogeneities within the same textural class can be discriminated using the index.

A replica of the framework of the textural problem is obtained when considering the evaluation of heterogeneity of biomass distributions in ecology. It is suggested how the use of balanced entropy may apply in this context.

Acknowledgements

Research partially supported by Plan Nacional de Investigación Científica, Desarrollo e Innovación Tecnológica (I+D+I) under ref. REN2000-1542 GLO, Spain and by Comunidad de Madrid under ref. 07M/0048/2000.

References

Baver, L.D., Gardner, W.H., Gardner, W.R., 1972. Soil Physics, 4th ed., John Wiley & Sons.

Caniego, J., Martín, M.A., San José, F., 2001. Singularity features of pore-size distribution in soil: singularity strength analysis and entropy spectrum. Fractals 9 (3), 305–316.

Cover, T., Thomas, J.A., 1991. Elements of Information Theory, John Wiley & Sons.

Damuth, J., 1981. Population density and body size in mammals. Nature 290, 699–700.

Damuth, J., 1987. Interspecific allometry of population density in mammals and other animals: the independence of body mass and population energy use. Biol. J. Linn. Soc. 31, 193–246.

Folk, R.L., 1954. The distinction between grain size and mineral composition in sedimentary rock nomenclature. J. Geol. 62, 344–359.

Ibañez, J.J., De-Alba, S., 2000. Pedodiversity and scaling laws: sharing Martín and Rey's opinion on the role of the Shannon's index as a measure of diversity. Geoderma 98, 5–9.

Ibañez, J.J., De-Alba, S., Lobo, A., Zucarello, V., 1998. Pedodiversity and global soil patterns at coarse scales (with discussion). Geoderma 83, 71–192.

Kullback, S., Leibler, R.A., 1951. On information and sufficiency. Ann. Math. Stat. 22, 79–86.

Lurié, D., Valls, J., Wagensberg, J., 1983. Thermodynamic approach to biomass distribution in ecological systems. Bull. Math. Biol. 45 (5), 869–872.

Margalef, R., 1958. Information theory in ecology. Gen. Syst. 3, 36–71.

Martín, M.A., Montero, E., 2001. Laser diffraction and multifractal analysis for the characterization of dry volume–size soil distributions. Soil Tillage Res. 64 (1–2), 113–123.

Martín, M.A., Rey, J.-M., 2000. On the role of Shannon's entropy as a measure of heterogeneity. Geoderma 98, 1–3.

Martín, M.A., Rey, J.-M., 2002. A balanced-entropy index to evaluate heterogeneity of mass–size distributions. Physica A, submitted for publication.

Martín, M.A., Rey, J.-M., Taguas, F.J., 2001. An entropy-based parametrization of soil texture via fractal modelling of particle-size distribution. Proc. R. Soc. Lond. Ser. A 457, 937–948.

Martín, M.A., Taguas, F.J., 1998. Fractal modelling, characterization and simulation of particle-size distribution in soil. Proc. R. Soc. Lond. Ser. A 454, 1457–1468.

Renyi, A., 1957. In Trans. 2nd Prague Conference on Information Theory, Statistical Decision Functions and Random Processes, pp. 545–556.

Shannon, C.E., 1948a. A mathematical theory of communication I. Bell Syst. Tech. J. 27, 379–423.

Shannon, C.E., 1948b. A mathematical theory of communication II. Bell Syst. Tech. J. 27, 623–656.

Shepard, F.P., 1954. Nomenclature based on sand–silt–clay ratios. J. Sediment. Petrol. 24 (3), 151–158.

Shirazi, M.A., Boersma, L., 1984. A unifying quantitative analysis of soil texture. Soil Sci. Soc. Am. J. 48, 142–147.

Soil Conservation Service (SCS), 1975. Soil Taxonomy: A Basic System of Soil Classification for Making and Interpreting Soil Surveys, Agricultural Handbook. No. 436, USDA–SCS, USA Department of Agriculture, pp. 486–742.

Taguas, F.J., Martín, M.A., Perfect, E., 1999. Simulation and testing of self-similar structures for soil particle-size distributions using iterated function systems. Geoderma 88, 191–203.

Tokeshi, M., 1993. Species abundance patterns and community structure. in: Advances in Ecological Research, vol. 24, Academic Press, pp. 111–186.

Turcotte, D.L., 1986. Fractals and fragmentation. J. Geophys. Res. 91, 1921–1926.

Vanoni, V.A., 1984. Sedimentation Engineering. Sedimentation Committee, Hydraulic Division, American Society of Civil Engineers, New York.