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Lipschitz continuous dynamic programming with discount II[☆]

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Abstract

We construct an alternative theoretical framework for stochastic dynamic programming which allows us to replace concavity assumptions with more flexible Lipschitz continuous assumptions. This framework allows us to prove that the value function of stochastic dynamic programming problems with discount is Lipschitz continuous in the presence of nonconcavities in the data of the problem. Our method allows us to treat problems with noninterior optimal paths. We also describe a discretization algorithm for the numerical computation of the value function, and we obtain the rate of convergence of this algorithm.

Keywords: Dynamic programming; Nonconcavities; Renewable resources; Nonsmoothness; Increasing marginal returns

1. Introduction

In this paper we complete the treatment of problems of stochastic dynamic programming with discount in a framework of Lipschitz continuous hypothesis on the data of the problem.

Dynamic programming with discount provides a setting for the analysis of optimal intertemporal transfers of economic resources. There is assumed the existence of a central planner who tries to maximize, over all feasible currents $c_1, c_2, c_3 \dots$ of future consumptions, $\sum_{i=1}^{\infty} \beta^i ER(c_i)$, where $ER(c_i)$ is the expected return at period i derived from consumption c_i and $\beta \in (0, 1)$ is the discount factor (see Section 2 for a full exposition of the problem). Typically R is a monetary benefit or some subjective utility which summarizes the central planner's objective, and β reflects the willingness to substitute between present and future return. Some of the principal models in today's macroeconomic theory as described by Ljungqvist and Sargent [13] are expressible in this framework. Also, many problems at the microeconomic level are currently treated in this setting (see [19]), in particular, problems of optimal exploitation of renewable resources (see Example 10).

The standard theory of dynamic programming with discount relies heavily on the concavity of the data of the problem (i.e., state space, return function and technological constraint correspondence). It first requires compactness and continuity of the data in order to guarantee the existence, uniqueness and continuity of the value function. Concavity, smoothness and monotonicity are then required in order to guarantee the smoothness and numerical

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